Statistics

Lecture 12

Feb 19-8:47 AM

In a survey of 475 people, 32% of them had more than 2 credit cards.

$n = 475$

$\hat{p} = .32 \rightarrow x = n\hat{p} = 475(.32) = \boxed{152}$

if decimal → Round-up

Find 98% conf. interval for the prop. of all people that have more than 2 credit cards.

C-level: .98

1-Prop Z Int

$x = 152$

$n = 475$

C-level: .98

$\boxed{.270 < P < .370}$

We are 98% confident that between 27% & 37% of all people have more than 2 cc.

$E = \frac{.370 - .270}{2} = \boxed{.05}$

$\hat{P} = \frac{.370 + .270}{2} = \boxed{.32}$

Nov 14-6:51 PM

Suppose we wish to construct 99% conf. interval for the prop. of all people with more than 2 credit cards and we want the margin of error not to exceed 4%, what is the minimum sample size?

$n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$, if $\hat{p}$ & $\hat{q}$ are unknown $n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2$

If decimal ⇒ Round-up

$\hat{p} = .32$

$\hat{q} = 1 - \hat{p} = .68$

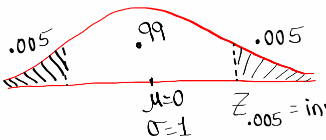
$E = 4\% = .04$

C-level: 99%

$n = (.32)(.68)\left(\frac{2.576}{.04}\right)^2$

$= 902.4655...$

$\boxed{n = 903}$



$Z_{.005} = \text{invNorm}(.995, 0, 1) = 2.576$

Nov 14-6:57 PM

75 of 320 students had a full-time job while going to school. $n = 320$ $x = 75$

use 95%

NO C-level

Find $\boxed{\text{conf. interval}}$ for the prop. of all students that have full-time job.

1 – PropZInt

$x = 75$

$n = 320$

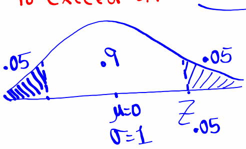
C-level: .95

$\boxed{.188 < p < .281}$

$E = \frac{.281 - .188}{2} = .047$

$\hat{p} = \frac{.281 + .188}{2} = .235$

Find minimum sample size needed if we wish to construct 90% C-level, and error not to exceed 5%. Assume $\hat{p}$ & $\hat{q}$ are unknown.



$Z_{.05} = \text{invNorm}(.95, 0, 1) = 1.645$

$n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2$

$= .25\left(\frac{1.645}{.05}\right)^2$

$= 270.6025$

$n \approx 271$

Nov 14-7:04 PM

In a Sample of 36 exams, the mean Score was 84.5.

$n = 36$

$\bar{x} = 84.5$

Find 99% Conf. interval for the mean Score of all exams assuming the Standard deviation of all exam Score is 9.8.

C-level: .99

$\sigma = 9.8$

$80.293 < \mu < 88.707$

Inpt: STATS

If σ Known ⇒ Z Interval

If σ unknown ⇒ T Interval

σ: 9.8

$\bar{x} = 84.5$ ← 1-decimal

$n = 36$

C-level: .99

$\boxed{80.3 < \mu < 88.7}$

$E = \frac{88.7 - 80.3}{2} = \boxed{4.2}$

$\bar{x} = \frac{88.7 + 80.3}{2} = \boxed{84.5}$

we are 99% Confident that the mean of all Scores is between 80.3 & 88.7

Nov 14-7:15 PM

18 Students were randomly Selected, their mean age was 34 with standard deviation 8.

$n = 18$, $\bar{x} = 34$, $S = 8$

Find Conf. interval for the mean age of all Students.

↳ No C-level
⇒ Use .95

$30.022 < \mu < 37.978$

$\boxed{30 < \mu < 38}$

If σ Known → Use Z Interval

If σ Unknown → Use T Interval

inpt: Stats

$\bar{x} = 34$ ← whole #

$S = 8$

$n = 18$

C-level: .95

$E = \frac{38 - 30}{2} = 4$, $\bar{x} = \frac{38 + 30}{2} = \boxed{34}$

Nov 14-7:25 PM

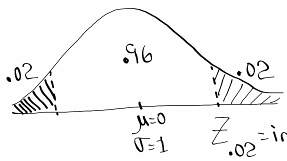
Given C-level: .96, $E=8$, $\sigma=15$

find minimum Sample Size needed To construct conf. interval for population mean.

$$n=\left(\frac{Z_{\alpha/2}\cdot\sigma}{E}\right)^2 \qquad n=\left(\frac{2.054\cdot 15}{8}\right)^2 = 21.358$$

when decimal → Round-up

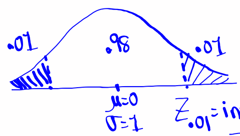
$\boxed{n=22}$



$Z_{.02}=\text{invNorm}(.98,0,1)=\boxed{2.054}$

what if we want $\boxed{98\%\text{ C-level}}$ and error not to exceed 5.

$$n=\left(\frac{Z_{\alpha/2}\cdot\sigma}{E}\right)^2$$



$Z_{.01}=\text{invNorm}(.99,0,1)=\boxed{2.326}$

$$\left(\frac{2.326\cdot 15}{5}\right)^2 = 48.692\ldots$$

$\boxed{n=49}$

Nov 14-7:32 PM

I randomly Selected 12 nurses. Here are their ages:

32	40	45	30
28	35	42	50
55	38	48	62

Find
1) $\bar{x}\approx 42$
2) $S\approx 10$

Round to whole #

Find 90% Conf. interval for the mean age of all nurses.

C-level: .9

σ unknown ⇒ T Interval

inpt: $\boxed{\text{STATS}}$

$\bar{x}=42$
$S=10$
$n=12$
C-level: .9

$36.816<\mu<47.184$

whole #

$\boxed{37<\mu<47}$

$$E=\frac{47-37}{2}=\boxed{5}$$

$$\bar{x}=\frac{47+37}{2}=\boxed{42}$$

Nov 14-7:42 PM

Find minimum Sample Size need to Construct 99% Conf. interval for the mean age of all nurses and error not to exceed 8 Yrs.

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

If σ is unknown use S.

$$= \left(\frac{2.576 \cdot 10}{8}\right)^2$$

$= 10.3684$

$\boxed{n \approx 11}$

$Z_{.005} = \text{invNorm}(.995, 0, 1)$

$= 2.576$

Redo for E = 4 Yrs.

$$n = \left(\frac{2.576 \cdot 10}{4}\right)^2 = 41.4736 \qquad \boxed{n \approx 42}$$

Nov 14-7:50 PM

t - Dist.

Graph is Symmetric and bell-Shape.

Total area = 1

It comes with degrees of freedom.

$\mu = 0$, σ unknown

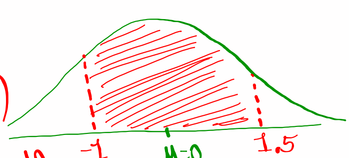
Find $P(-1 < t < 1.5)$ with df = 9.

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$

tcdf(-1 , 1.5 , 9)

lower upper df

$= \boxed{.744}$



Nov 14-8:12 PM

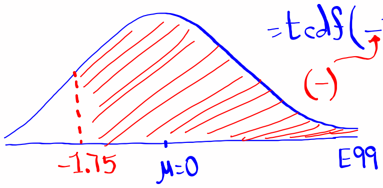
find $P(t > -1.75)$ with $df = 14$.

lower upper df

$= \text{tcdf}(-1.75, E99, 14)$

(-)

$\boxed{7}$ $\boxed{2nd}\boxed{,}$

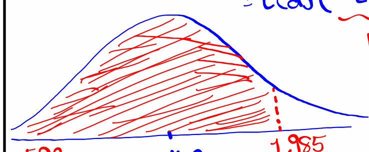


$= \boxed{.949}$

find $P(t < 1.985)$ with $df = 19$.

$= \text{tcdf}(\underbrace{-E99}_{L}, \underbrace{1.985}_{U}, \underbrace{19}_{df})$

$= \boxed{.969}$

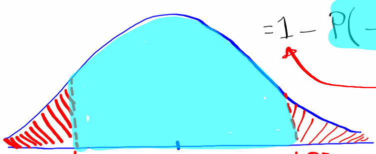


Nov 14-8:16 PM

find $P(t < -1.5 \text{ OR } t > 1.85)$ with $df = 11$

$= 1 - P(-1.5 < t < 1.85)$

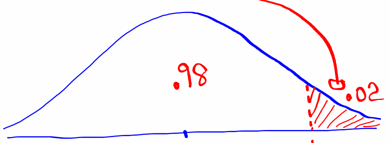
Total Area



$= 1 - \text{tcdf}(-1.5, 1.85, 11) = \boxed{.127}$

Nov 14-8:22 PM

Find $t_{.02}$ with $df = 8$



$\mu = 0$

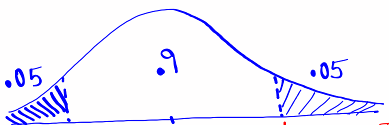
σ unknown

Left Area   df

$t_{.02} = invT(.98, 8)$

$= \boxed{2.449}$

Find $t_{\alpha/2}$ for $\boxed{90\% \text{ C-level}}$ with $df = 10$.



$\mu = 0$

σ unknown

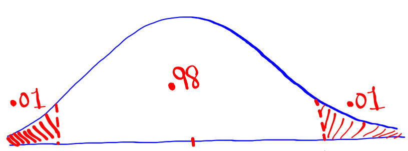
$t_{.05} = invT(.95, 10) = \boxed{1.812}$

Nov 14-8:26 PM

Find $\pm t_{\alpha/2}$ with $\alpha = .02$ and $df = 9$.

$\alpha/2 = .02/2 = .01$

$1 - \alpha = .98$



$-t_{.01}$

$\mu = 0$

σ unknown

$t_{.01} = invT(.99, 9)$

$= \boxed{2.821}$

Nov 14-8:31 PM

What is degrees of freedom?

10 People at work, and You bring 10 Donuts.

First Person — 10 choices
Second 〃 — 9 choices
Third 〃 — 8 〃
⋮
Last Person — No choice

degrees of freedom = 9

7 clean shirt
You were 1 clean shirt per day.

Monday — 7 choices
Tuesday — 6 〃
Wednesday — 5 〃
⋮
Sunday — No choice

$df = 7 - 1$
$= 6$

$\boxed{\text{when Constructing Conf. interval for Pop. mean and } \sigma \text{ unknown} \quad df = n - 1}$

As df increases $\Rightarrow t_{\alpha/2} \approx z_{\alpha/2}$

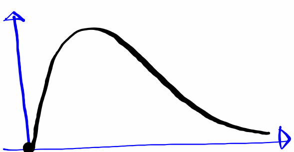
Nov 14-8:34 PM

Chi-Square Dist: χ^2

Graph begins at 0, Skewed to the right.

Not Symmetric but total Area = 1

It comes with df.

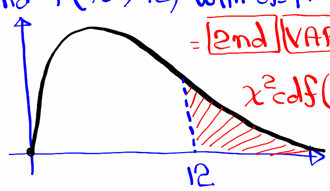


Find $P(\chi^2 > 12)$ with $df = 9$

$= \boxed{2nd}\boxed{VARS}$

$\chi^2cdf(12, E99, 9) = \boxed{.213}$

lower — Upper — df

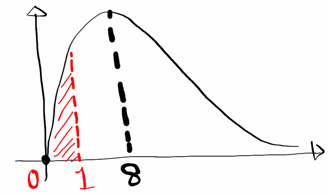


Nov 14-8:42 PM

Find $P(\chi^2 < 1)$ with $df = 8$.

$= \chi^2cdf(0, 1, 8)$

$= \boxed{.002}$

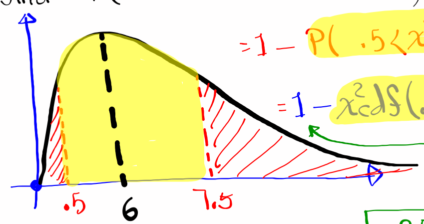


Find $P(\chi^2 < .5 \text{ OR } \chi^2 > 7.5)$ with $df = 6$.

$= 1 - P(.5 < \chi^2 < 7.5)$

$= 1 - \chi^2cdf(.5, 7.5, 6)$

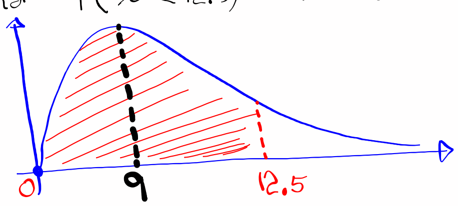
Total Area = 1



$= \boxed{.279}$

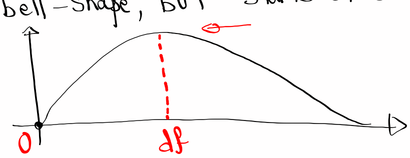
Nov 14-8:46 PM

Find $P(\chi^2 < 12.5)$ with $df = 9$.



$= \chi^2cdf(0, 12.5, 9) = \boxed{.843}$

As df increase, Chi-square dist graph looks bell-shape, but starts at 0.



Nov 14-8:53 PM

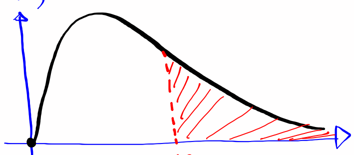
F – Dist

Graph is similar to χ^2 – Dist.

It comes with two df.

Ndf & Ddf

Find $P(F > 10)$ with Ndf = 4 & Ddf = 20.



2nd VARS

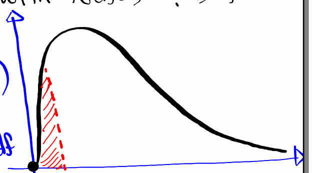
Fcdf(10, E99, 4 , 20) = $\boxed{1.298 \times 10^{-4}}$

Find $P(F < 1.75)$ with Ndf = 5 & Ddf = 25

Fcdf(0 , 1.75 , 5 , 25)

L U Ndf Ddf

= $\boxed{.840}$



Nov 14-8:58 PM

Find $P(F < 2$ and $F > 10)$ with Ndf = 4, Ddf = 30.

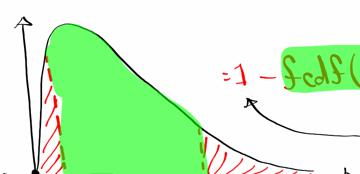
Details Matter

No overlap → Impossible event

$P(F < 2$ OR $F > 10)$ with Ndf = 4, Ddf = 30

= 1 – Fcdf(2, 10, 4, 30)

Total Area



= $\boxed{.880}$

Nov 14-9:08 PM